ME 141

Engineering Mechanics

Lecture 11: Kinetics of particles: Energy method

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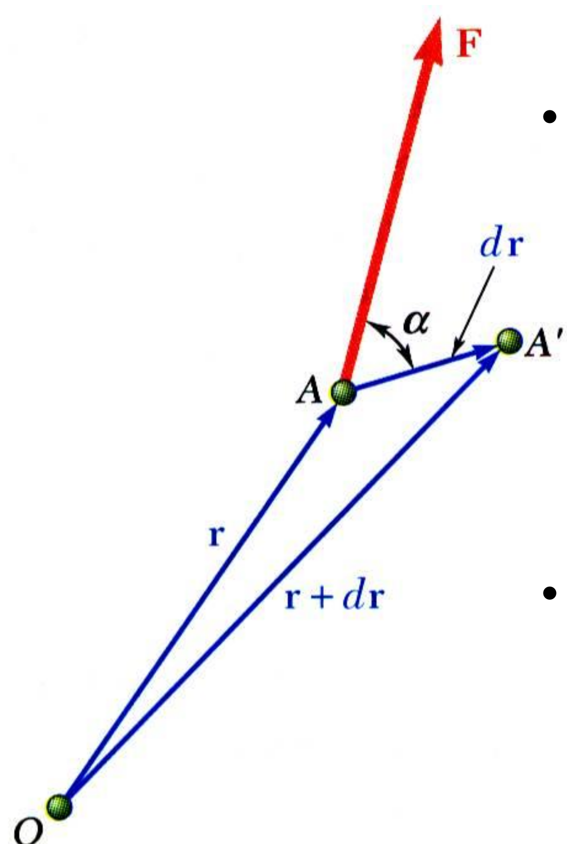
Courtesy: Vector Mechanics for Engineers, Beer and Johnston

Introduction

Forces and Accelerations	Velocities and Displacements	Velocities and Time
↓	↓	↓
Newton's Second Law (last chapter)	**Work-Energy**	**Impulse-Momentum**
$\sum \vec{F} = m\vec{a}_G$	$T_1 + U_{1\to 2} = T_2$	$m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}\, dt = m\vec{v}_2$

Work of a Force



- Differential vector $d\vec{r}$ is the *particle displacement*.
- *Work of the force* is

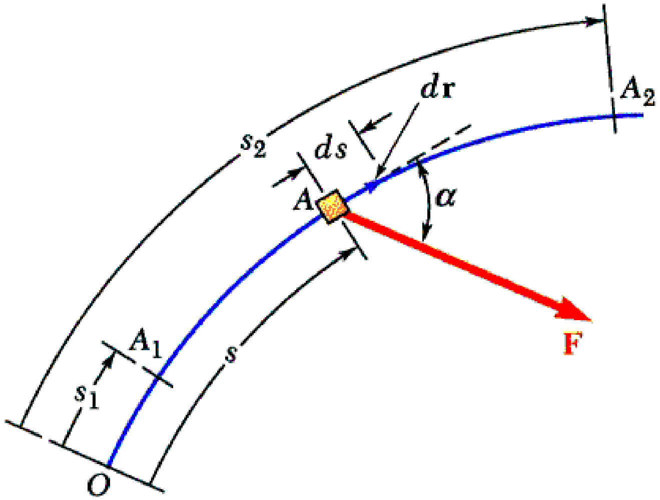
$$dU = \vec{F} \bullet d\vec{r}$$
$$= F\,ds\ \cos\alpha$$
$$= F_x dx + F_y dy + F_z dz$$

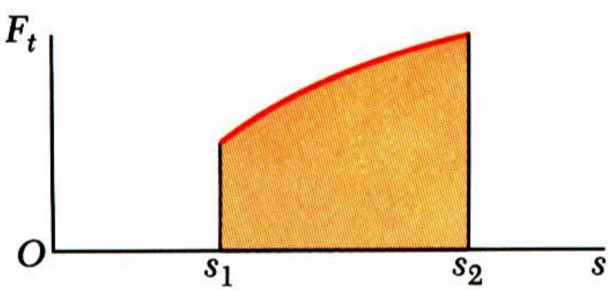
- Work is a ***scalar*** quantity, i.e., it has magnitude and sign but not direction.

- Dimensions of work are length × force. Units are

$$1\,\text{J}\,(\textit{joule}) = (1\,\text{N})(1\,\text{m}) \qquad 1\,\text{ft} \cdot \text{lb} = 1.356\,\text{J}$$

Work of a Force





- Work of a force during a finite displacement,

$$U_{1\to 2} = \int_{A_1}^{A_2} \vec{F} \bullet d\vec{r}$$

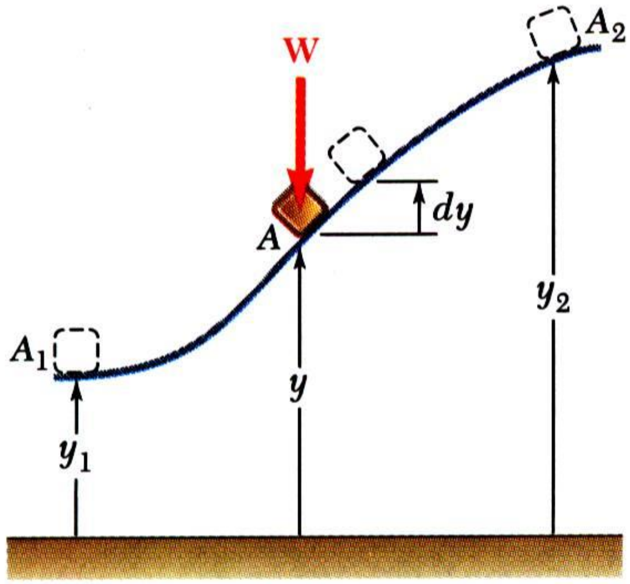
$$= \int_{s_1}^{s_2} (F\cos\alpha)ds = \int_{s_1}^{s_2} F_t \; ds$$

$$= \int_{A_1}^{A_2} \left(F_x dx + F_y dy + F_z dz\right)$$

- Work is represented by the area under the curve of F_t plotted against s.

- F_t is the force in the direction of the displacement ds

Work of a Force



- Work of the force of gravity,

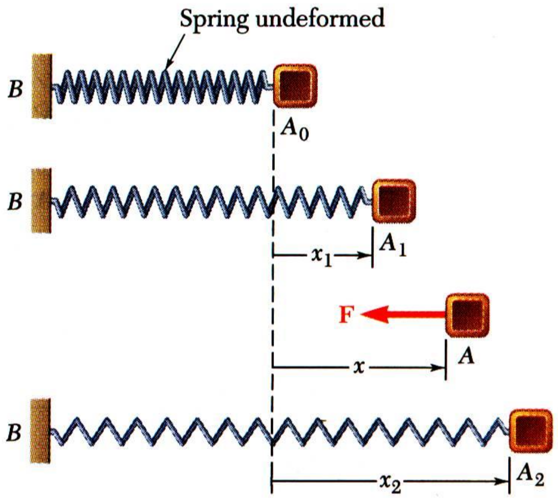
$$dU = F_x dx + F_y dy + F_z dz$$
$$= -W\ dy$$
$$U_{1\to 2} = -\int_{y_1}^{y_2} W\ dy$$
$$= -W(y_2 - y_1) = -W\ \Delta y$$

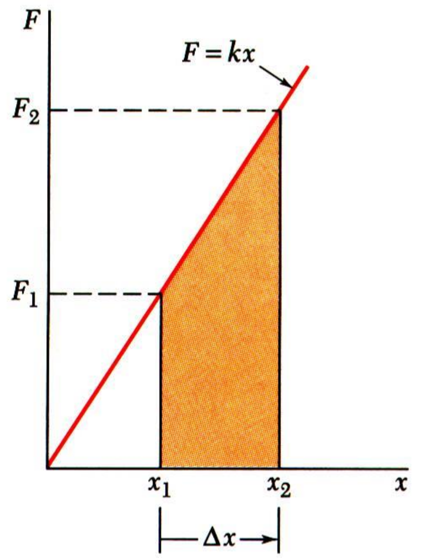
- Work *of the weight* is equal to product of weight *W* and vertical displacement Δy.

- **In the figure above, when is the work done by the weight positive?**

a) Moving from y_1 to y_2 b) Moving from y_2 to y_1 c) Never

Work of a Force





- Magnitude of the force exerted by a spring is proportional to deflection,

$$F = kx$$

$$k = \text{spring constant (N/m or lb/in.)}$$

- Work *of the force exerted by spring*,

$$dU = -F\,dx = -kx\,dx$$

$$U_{1\to 2} = -\int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

- Work *of the force exerted by spring* is positive when $x_2 < x_1$, i.e., when the spring is returning to its undeformed position.

- Work of the force exerted by the spring is equal to negative of area under curve of F plotted against x,

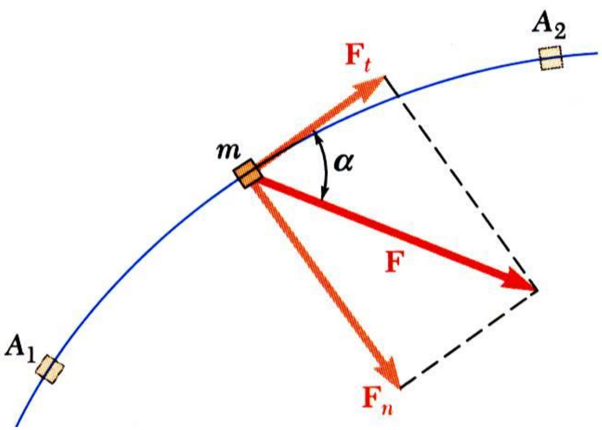
$$U_{1\to 2} = -\tfrac{1}{2}(F_1 + F_2)\,\Delta x$$

Work of a Force

Forces which *do not* do work ($ds = 0$ or $\cos \alpha = 0$):

- Reaction at frictionless pin supporting rotating body,
- Reaction at frictionless surface when body in contact moves along surface,
- Reaction at a roller moving along its track, and
- Weight of a body when its center of gravity moves horizontally.

Particle Kinetic Energy: Principle of Work & Energy



- Consider a particle of mass m acted upon by force $\vec{F}$

$$F_t = ma_t = m\frac{dv}{dt}$$

$$= m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{dv}{ds}$$

$$F_t\, ds = mv\, dv$$

- Integrating from A_1 to A_2 ,

$$\int_{s_1}^{s_2} F_t ds = m\int_{v_1}^{v_2} v\, dv = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

$$\boxed{U_{1\to 2} = T_2 - T_1} \qquad T = \tfrac{1}{2}mv^2 = \textit{kinetic energy}$$

- *The work of the force $\vec{F}$ is equal to the change in kinetic energy of the particle.*

- Units of work and kinetic energy are the same:

$$T = \tfrac{1}{2}mv^2 = \mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = \left(\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right)\mathrm{m} = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

Power and Efficiency

- $Power =$ rate at which work is done.

$$= \frac{dU}{dt} = \frac{\vec{F} \bullet d\vec{r}}{dt}$$

$$= \vec{F} \bullet \vec{v}$$

- Dimensions of power are work/time or force*velocity. Units for power are

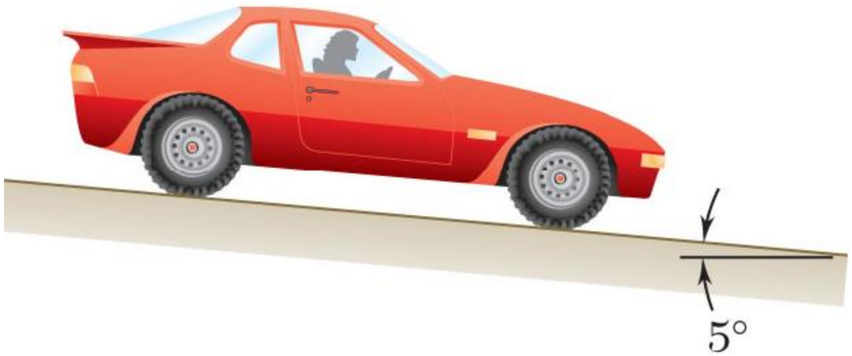
$$1\,\mathrm{W}\ (\mathrm{watt}) = 1\frac{\mathrm{J}}{\mathrm{s}} = 1\,\mathrm{N}\cdot\frac{\mathrm{m}}{\mathrm{s}} \quad \text{or} \quad 1\,\mathrm{hp} = 550\frac{\mathrm{ft}\cdot\mathrm{lb}}{\mathrm{s}} = 746\,\mathrm{W}$$

- $\eta =$ efficiency

$$= \frac{\text{output work}}{\text{input work}}$$

$$= \frac{\text{power output}}{\text{power input}}$$

Sample Problem 13.1



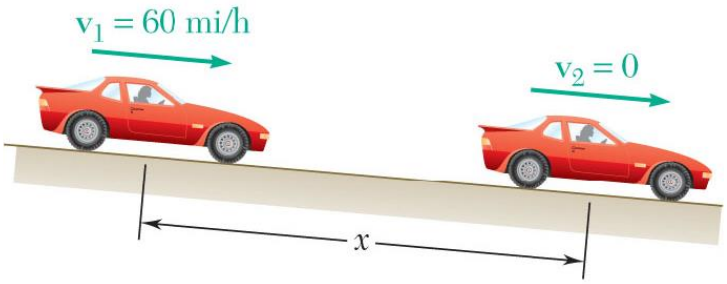
An automobile weighing 4000 lb is driven down a 5° incline at a speed of 60 mi/h when the brakes are applied causing a constant total breaking force of 1500 lb.

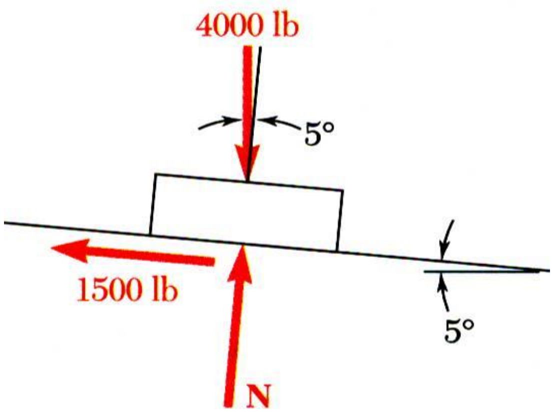
Determine the distance traveled by the automobile as it comes to a stop.

SOLUTION:

- Evaluate the change in kinetic energy.
- Determine the distance required for the work to equal the kinetic energy change.

Sample Problem 13.1





SOLUTION:

- Evaluate the change in kinetic energy.

$$v_1 = \left(60\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\text{ ft}}{\text{mi}}\right)\left(\frac{\text{h}}{3600\text{ s}}\right) = 88\text{ ft/s}$$

$$T_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(4000/32.2)(88)^2 = 481000\text{ ft}\cdot\text{lb}$$

$$v_2 = 0 \qquad T_2 = 0$$

- Determine the distance required for the work to equal the kinetic energy change.

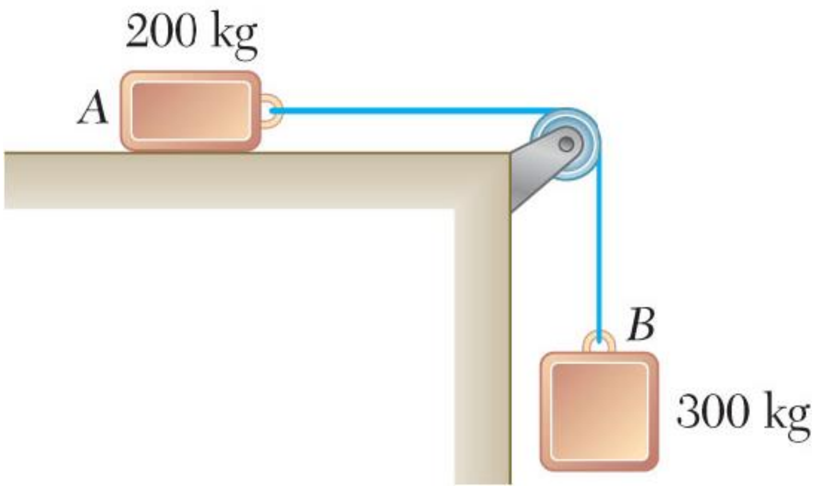
$$U_{1\to2} = (-1500\text{ lb})x + (4000\text{ lb})(\sin 5°)x$$
$$= -(1151\text{ lb})x$$

$$T_1 + U_{1\to2} = T_2$$

$$481000\text{ ft}\cdot\text{lb} - (1151\text{ lb})x = 0$$

$$\boxed{x = 418\text{ ft}}$$

Sample Problem 13.2

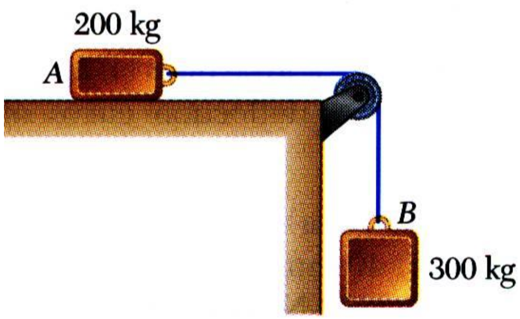


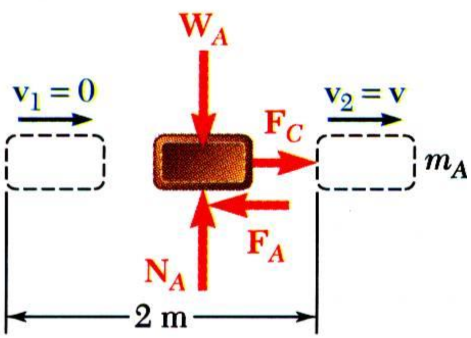
Two blocks are joined by an inextensible cable as shown. If the system is released from rest, determine the velocity of block *A* after it has moved 2 m. Assume that the coefficient of friction between block *A* and the plane is $\mu_k = 0.25$ and that the pulley is weightless and frictionless.

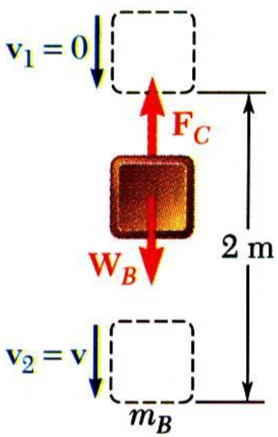
SOLUTION:

- Apply the principle of work and energy separately to blocks *A* and *B*.
- When the two relations are combined, the work of the cable forces cancel. Solve for the velocity.

Sample Problem 13.2







SOLUTION:

- Apply the principle of work and energy separately to blocks *A* and *B*.

$$W_A = (200\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right) = 1962\,\mathrm{N}$$

$$F_A = \mu_k N_A = \mu_k W_A = 0.25(1962\,\mathrm{N}) = 490\,\mathrm{N}$$

$$T_1 + U_{1\to 2} = T_2 :$$

$$0 + F_C(2\,\mathrm{m}) - F_A(2\,\mathrm{m}) = \tfrac{1}{2} m_A v^2$$

$$F_C(2\,\mathrm{m}) - (490\,\mathrm{N})(2\,\mathrm{m}) = \tfrac{1}{2}(200\,\mathrm{kg})v^2$$

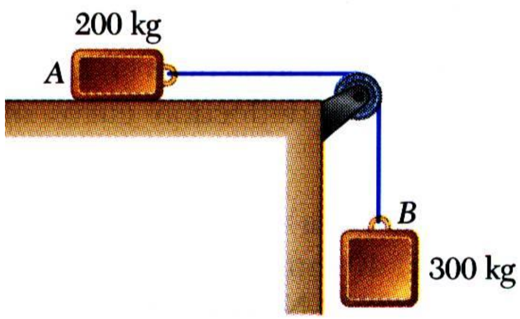
$$W_B = (300\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right) = 2940\,\mathrm{N}$$

$$T_1 + U_{1\to 2} = T_2 :$$

$$0 - F_c(2\,\mathrm{m}) + W_B(2\,\mathrm{m}) = \tfrac{1}{2} m_B v^2$$

$$-F_c(2\,\mathrm{m}) + (2940\,\mathrm{N})(2\,\mathrm{m}) = \tfrac{1}{2}(300\,\mathrm{kg})v^2$$

Sample Problem 13.2



- When the two relations are combined, the work of the cable forces cancel. Solve for the velocity.

$$F_C(2\,\text{m}) - (490\,\text{N})(2\,\text{m}) = \tfrac{1}{2}(200\,\text{kg})v^2$$

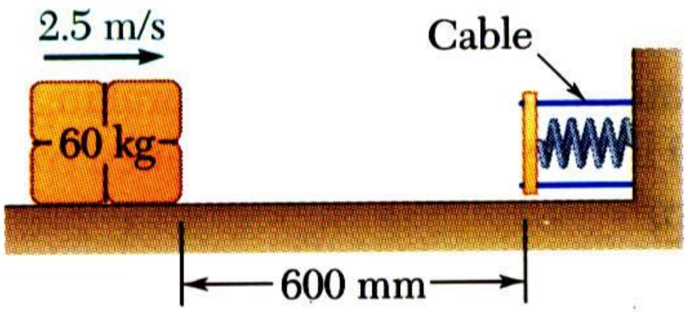
$$-F_c(2\,\text{m}) + (2940\,\text{N})(2\,\text{m}) = \tfrac{1}{2}(300\,\text{kg})v^2$$

$$(2940\,\text{N})(2\,\text{m}) - (490\,\text{N})(2\,\text{m}) = \tfrac{1}{2}(200\,\text{kg} + 300\,\text{kg})v^2$$

$$4900\,\text{J} = \tfrac{1}{2}(500\,\text{kg})v^2$$

$$\boxed{v = 4.43\,\text{m/s}}$$

Sample Problem 13.3



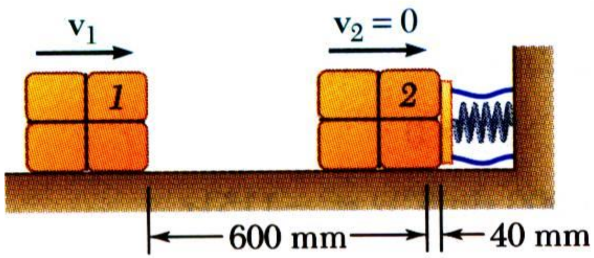
A spring is used to stop a 60 kg package which is sliding on a horizontal surface. The spring has a constant k = 20 kN/m and is held by cables so that it is initially compressed 120 mm. The package has a velocity of 2.5 m/s in the position shown and the maximum deflection of the spring is 40 mm.

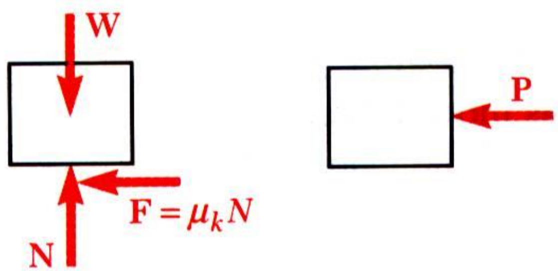
Determine *(a)* the coefficient of kinetic friction between the package and surface and *(b)* the velocity of the package as it passes again through the position shown.

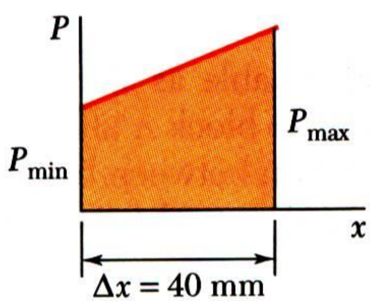
SOLUTION:

- Apply the principle of work and energy between the initial position and the point at which the spring is fully compressed and the velocity is zero. The only unknown in the relation is the friction coefficient.
- Apply the principle of work and energy for the rebound of the package. The only unknown in the relation is the velocity at the final position.

Sample Problem 13.3







SOLUTION:

- Apply principle of work and energy between initial position and the point at which spring is fully compressed.

$$T_1 = \tfrac{1}{2} m v_1^2 = \tfrac{1}{2}(60\,\text{kg})(2.5\,\text{m/s})^2 = 187.5\,\text{J} \qquad T_2 = 0$$

$$(U_{1\to2})_f = -\mu_k W x$$

$$= -\mu_k (60\,\text{kg})(9.81\,\text{m/s}^2)(0.640\,\text{m}) = -(377\,\text{J})\mu_k$$

$$P_{min} = k x_0 = (20\,\text{kN/m})(0.120\,\text{m}) = 2400\,\text{N}$$

$$P_{max} = k(x_0 + \Delta x) = (20\,\text{kN/m})(0.160\,\text{m}) = 3200\,\text{N}$$

$$(U_{1\to2})_e = -\tfrac{1}{2}(P_{min} + P_{max})\Delta x$$

$$= -\tfrac{1}{2}(2400\,\text{N} + 3200\,\text{N})(0.040\,\text{m}) = -112.0\,\text{J}$$

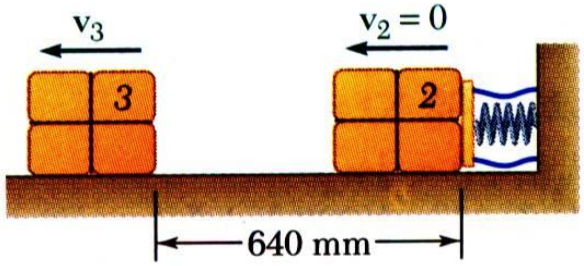
$$U_{1\to2} = (U_{1\to2})_f + (U_{1\to2})_e = -(377\,\text{J})\mu_k - 112\,\text{J}$$

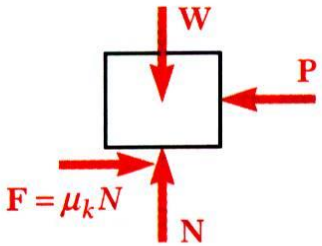
$$T_1 + U_{1\to2} = T_2 :$$

$$187.5\,\text{J} - (377\,\text{J})\mu_k - 112\,\text{J} = 0$$

$$\boxed{\mu_k = 0.20}$$

Sample Problem 13.3





- Apply the principle of work and energy for the rebound of the package.

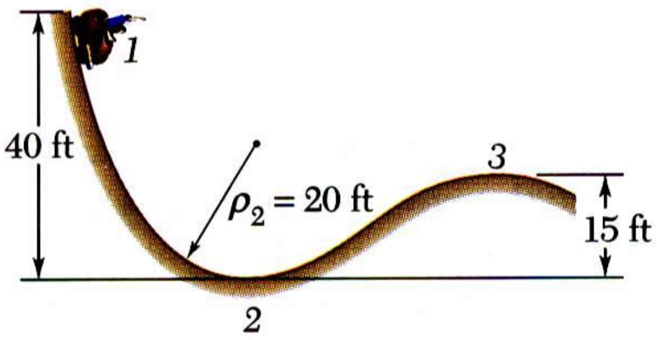
$$T_2 = 0 \qquad T_3 = \tfrac{1}{2} m v_3^2 = \tfrac{1}{2}(60\,\text{kg}) v_3^2$$

$$U_{2\to3} = (U_{2\to3})_f + (U_{2\to3})_e = -(377\ \text{J})\mu_k + 112\ \text{J}$$
$$= +36.5\ \text{J}$$

$$T_2 + U_{2\to3} = T_3 :$$
$$0 + 36.5\,\text{J} = \tfrac{1}{2}(60\,\text{kg}) v_3^2$$

$$\boxed{v_3 = 1.103\,\text{m/s}}$$

Sample Problem 13.4



A 2000 lb car starts from rest at point 1 and moves without friction down the track shown.

Determine:

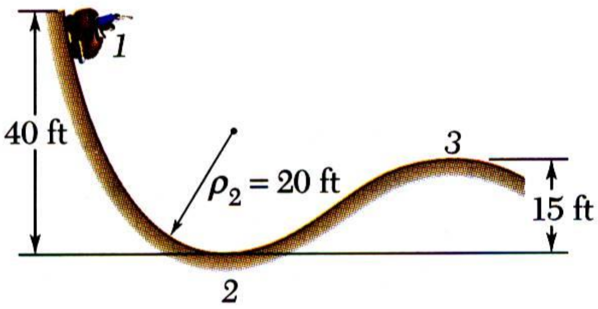
a) the force exerted by the track on the car at point 2, and

b) the minimum safe value of the radius of curvature at point 3.

SOLUTION:

- Apply principle of work and energy to determine velocity at point 2.
- Apply Newton's second law to find normal force by the track at point 2.
- Apply principle of work and energy to determine velocity at point 3.
- Apply Newton's second law to find minimum radius of curvature at point 3 such that a positive normal force is exerted by the track.

Sample Problem 13.4



SOLUTION:

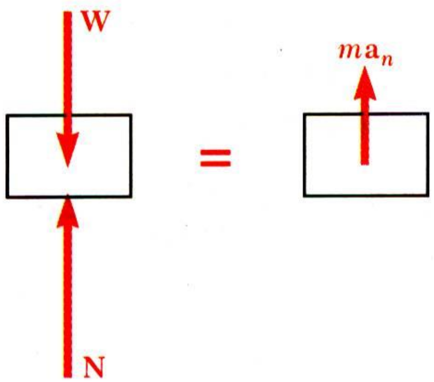
- Apply principle of work and energy to determine velocity at point 2.

$$T_1 = 0 \qquad T_2 = \tfrac{1}{2} m v_2^2 = \frac{1}{2}\frac{W}{g} v_2^2$$

$$U_{1\to 2} = +W(40\,\text{ft})$$

$$T_1 + U_{1\to 2} = T_2 : \qquad 0 + W(40\,\text{ft}) = \frac{1}{2}\frac{W}{g} v_2^2$$

$$v_2^2 = 2(40\,\text{ft})g = 2(40\,\text{ft})\left(32.2\,\text{ft}/s^2\right) \qquad v_2 = 50.8\,\text{ft}/\text{s}$$



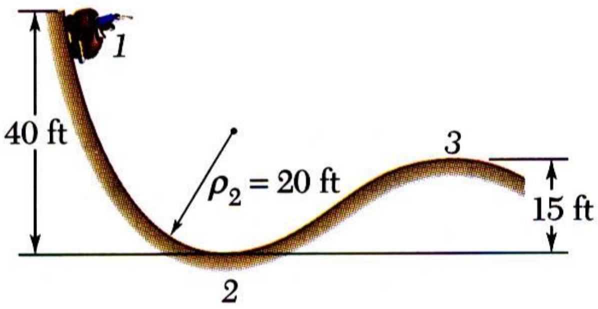
- Apply Newton's second law to find normal force by the track at point 2.

$$+\uparrow \sum F_n = m a_n :$$

$$-W + N = m a_n = \frac{W}{g}\frac{v_2^2}{\rho_2} = \frac{W}{g}\frac{2(40\,\text{ft})g}{20\,\text{ft}}$$

$$N = 5W \qquad \boxed{N = 10000\,\text{lb}}$$

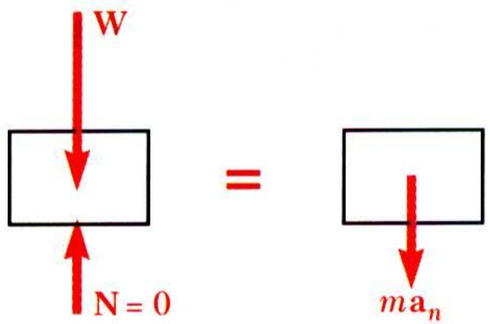
Sample Problem 13.4



- Apply principle of work and energy to determine velocity at point 3.

$$T_1 + U_{1\to 3} = T_3 \qquad 0 + W(25\,\text{ft}) = \frac{1}{2}\frac{W}{g}v_3^2$$

$$v_3^2 = 2(25\,\text{ft})g = 2(25\,\text{ft})(32.2\,\text{ft/s}) \qquad v_3 = 40.1\,\text{ft/s}$$



- Apply Newton's second law to find minimum radius of curvature at point 3 such that a positive normal force is exerted by the track.

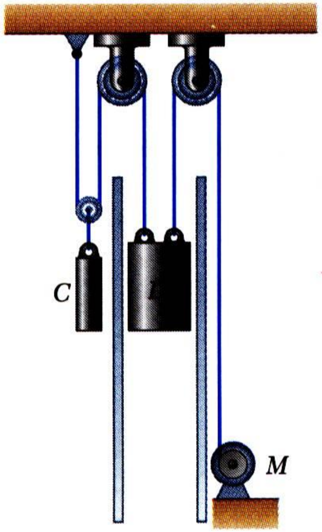
$$+\downarrow \sum F_n = m\,a_n :$$

$$W = m\,a_n$$

$$= \frac{W}{g}\frac{v_3^2}{\rho_3} = \frac{W}{g}\frac{2(25\,\text{ft})g}{\rho_3}$$

$$\boxed{\rho_3 = 50\,\text{ft}}$$

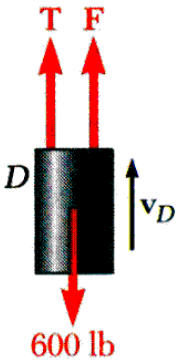
Sample Problem 13.5



The dumbwaiter *D* and its load have a combined weight of 600 lb, while the counterweight *C* weighs 800 lb.

Determine the power delivered by the electric motor *M* when the dumbwaiter *(a)* is moving up at a constant speed of 8 ft/s and *(b)* has an instantaneous velocity of 8 ft/s and an acceleration of 2.5 ft/s^2, both directed upwards.

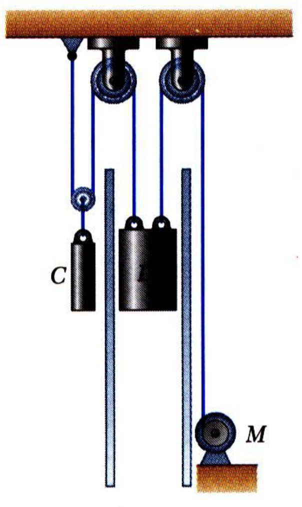
SOLUTION:



Force exerted by the motor cable has same direction as the dumbwaiter velocity. Power delivered by motor is equal to Fv_D, v_D = 8 ft/s.

- In the first case, bodies are in uniform motion. Determine force exerted by motor cable from conditions for static equilibrium.
- In the second case, both bodies are accelerating. Apply Newton's second law to each body to determine the required motor cable force.

Sample Problem 13.5



- In the first case, bodies are in uniform motion. Determine force exerted by motor cable from conditions for static equilibrium.

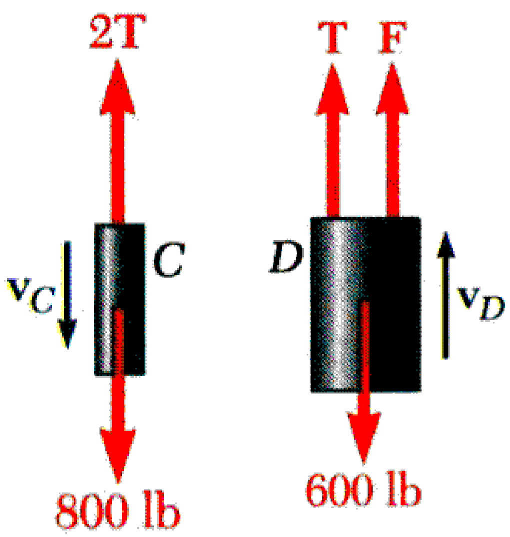
Free-body C:

$$+\uparrow \sum F_y = 0: \quad 2T - 800\,\text{lb} = 0 \qquad T = 400\,\text{lb}$$

Free-body D:

$$+\uparrow \sum F_y = 0: \quad F + T - 600\,\text{lb} = 0$$

$$F = 600\,\text{lb} - T = 600\,\text{lb} - 400\,\text{lb} = 200\,\text{lb}$$

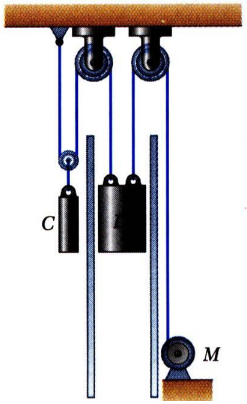


$$Power = Fv_D = (200\,\text{lb})(8\,\text{ft/s})$$

$$= 1600\,\text{ft}\cdot\text{lb/s}$$

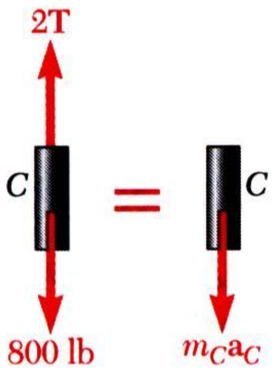
$$Power = (1600\,\text{ft}\cdot\text{lb/s})\frac{1\,\text{hp}}{550\,\text{ft}\cdot\text{lb/s}} = 2.91\,\text{hp}$$

Sample Problem 13.5



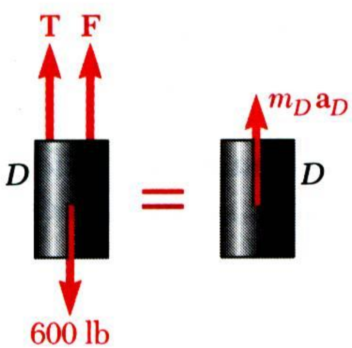
- In the second case, both bodies are accelerating. Apply Newton's second law to each body to determine the required motor cable force.

$$a_D = 2.5\,\text{ft/s}^2 \uparrow \qquad a_C = -\tfrac{1}{2}a_D = 1.25\,\text{ft/s}^2 \downarrow$$



Free-body C:

$$+\downarrow \sum F_y = m_C a_C: \quad 800 - 2T = \frac{800}{32.2}(1.25) \qquad T = 384.5\,\text{lb}$$



Free-body D:

$$+\uparrow \sum F_y = m_D a_D: \quad F + T - 600 = \frac{600}{32.2}(2.5)$$

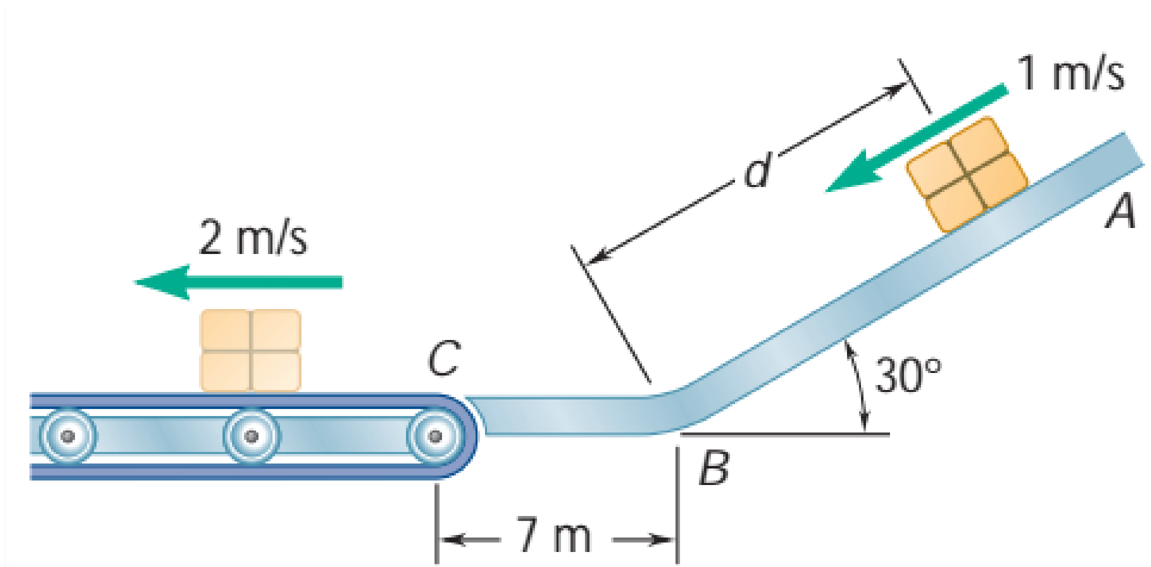
$$F + 384.5 - 600 = 46.6 \qquad F = 262.1\,\text{lb}$$

$$Power = Fv_D = (262.1\,\text{lb})(8\,\text{ft/s}) = 2097\,\text{ft}\cdot\text{lb/s}$$

$$Power = (2097\,\text{ft}\cdot\text{lb/s})\frac{1\,\text{hp}}{550\,\text{ft}\cdot\text{lb/s}} = 3.81\,\text{hp}$$

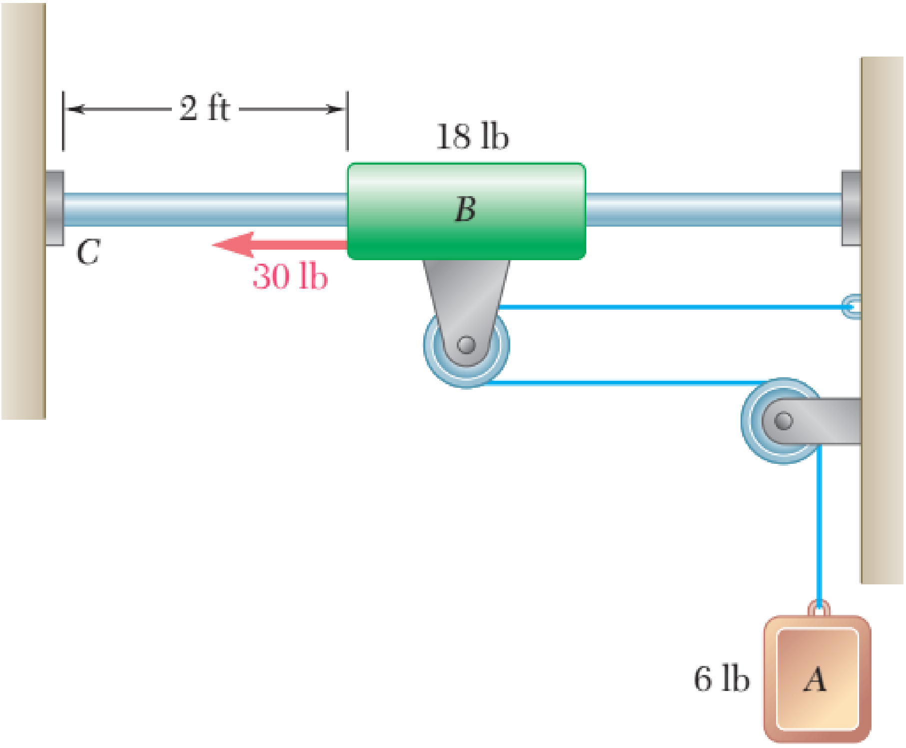
Prob # 13.11

Packages are thrown down an incline at *A* with a velocity of 1 m/s. The packages slide along the surface *ABC* to a conveyor belt which moves with a velocity of 2 m/s. Knowing that $\mu_k = 0.25$ between the packages and the surface *ABC*, determine the distance '*d*' if the packages are to arrive at *C* with a velocity of 2 m/s

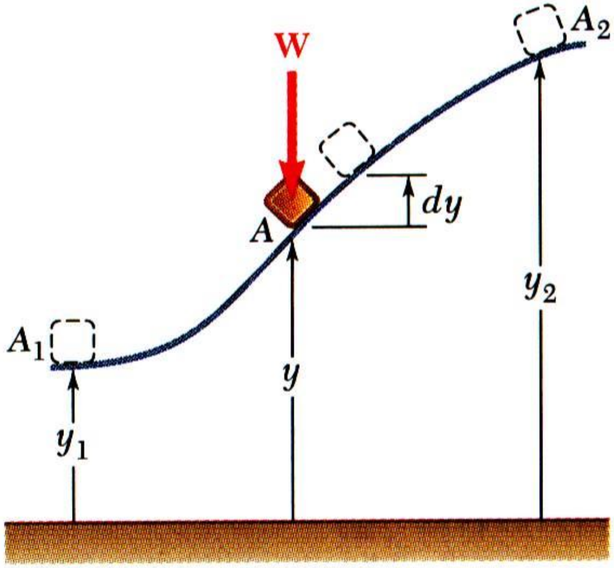


Prob # 13.20

The system shown is at rest when a constant 30-lb force is applied to collar *B*. (*a*) If the force acts through the entire motion, determine the speed of collar *B* as it strikes the support at *C*. (*b*) After what distance '*d*' should the 30-lb force be removed if the collar is to reach support *C* with zero velocity?



Potential Energy



- Work of the force of gravity $\vec{W}$,

  $$U_{1\to 2} = W\,y_1 - W\,y_2$$

- Work is independent of path followed; depends only on the initial and final values of Wy.

  $$V_g = Wy$$

  $=$ *potential energy* of the body with respect to *force of gravity*.

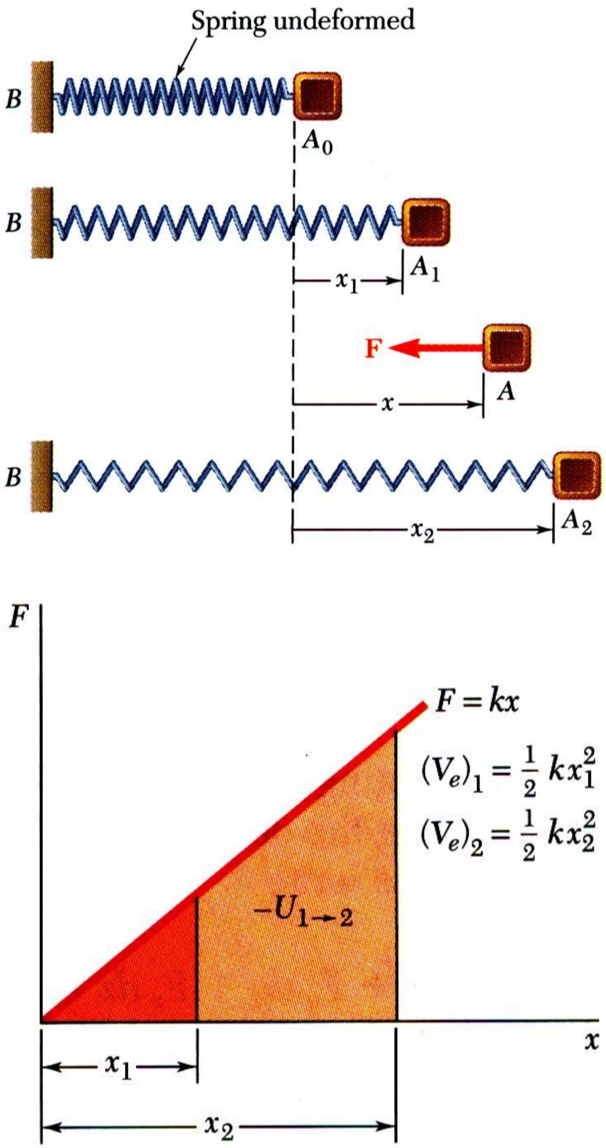
  $$U_{1\to 2} = \left(V_g\right)_1 - \left(V_g\right)_2$$

- Choice of datum from which the elevation y is measured is arbitrary.

- Units of work and potential energy are the same:

  $$V_g = Wy = \mathrm{N} \cdot \mathrm{m} = \mathrm{J}$$

Potential Energy



- Work of the force exerted by a spring depends only on the initial and final deflections of the spring,

$$U_{1\to2} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

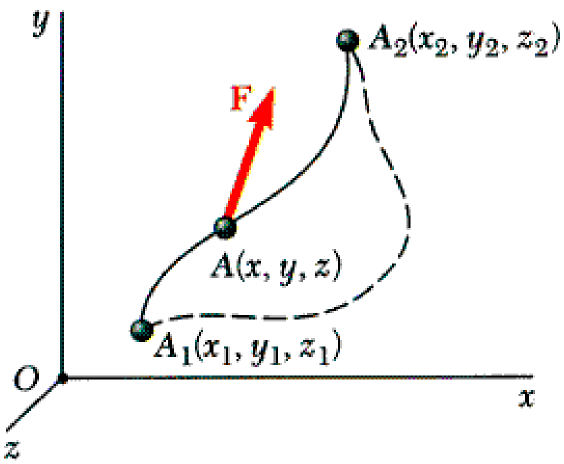
- The potential energy of the body with respect to the elastic force,

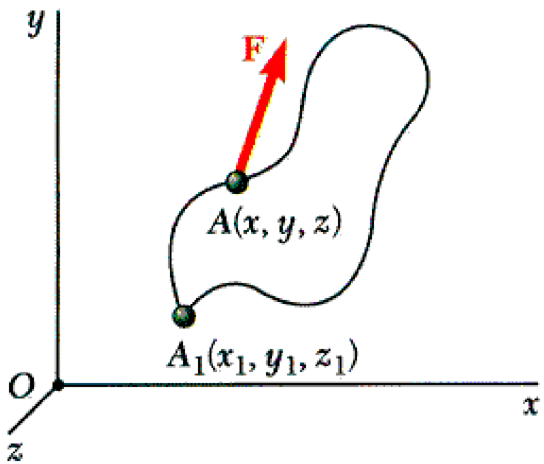
$$V_e = \tfrac{1}{2}kx^2$$

$$U_{1\to2} = (V_e)_1 - (V_e)_2$$

- Note that the preceding expression for V_e is valid only if the deflection of the spring is measured from its undeformed position.

Conservative Forces





- Concept of potential energy can be applied if the work of the force is independent of the path followed by its point of application.

  $$U_{1\to 2} = V(x_1, y_1, z_1) - V(x_2, y_2, z_2)$$

  Such forces are described as *conservative forces*.

- For any conservative force applied on a closed path,

  $$\oint \vec{F} \bullet d\vec{r} = 0$$

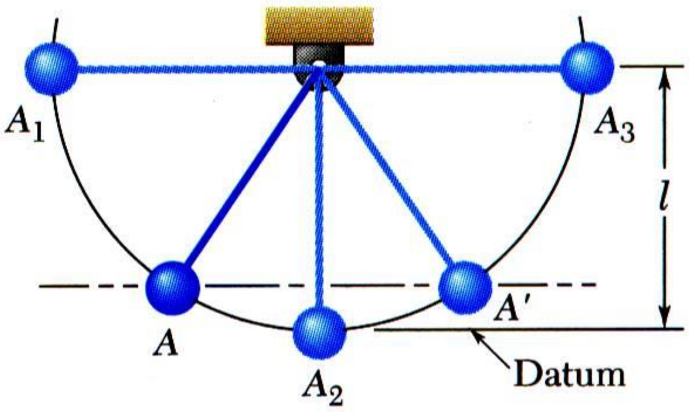
- Elementary work corresponding to displacement between two neighboring points,

  $$dU = V(x, y, z) - V(x + dx, y + dy, z + dz)$$
  $$= -dV(x, y, z)$$

  $$F_x dx + F_y dy + F_z dz = -\left(\frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz\right)$$

  $$\vec{F} = -\left(\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}\right) = -\mathbf{grad}\, V$$

Conservation of Energy



- Work of a conservative force,

$$U_{1\to 2} = V_1 - V_2$$

- Concept of work and energy,

$$U_{1\to 2} = T_2 - T_1$$

- Follows that

$$T_1 + V_1 = T_2 + V_2$$
$$E = T + V = \text{constant}$$

$$T_1 = 0 \quad V_1 = W\ell$$
$$T_1 + V_1 = W\ell$$

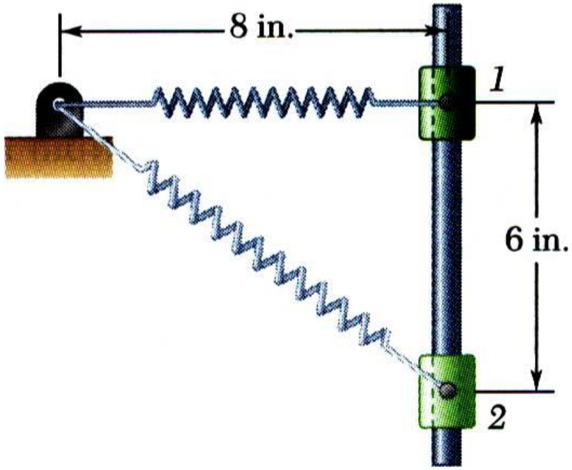
- When a particle moves under the action of conservative forces, the total mechanical energy is constant.

$$T_2 = \tfrac{1}{2} m v_2^2 = \frac{1}{2}\frac{W}{g}(2g\ell) = W\ell \quad V_2 = 0$$
$$T_2 + V_2 = W\ell$$

- Friction forces are not conservative. Total mechanical energy of a system involving friction decreases.

- Mechanical energy is dissipated by friction into thermal energy. Total energy is constant.

Sample Problem 13.6



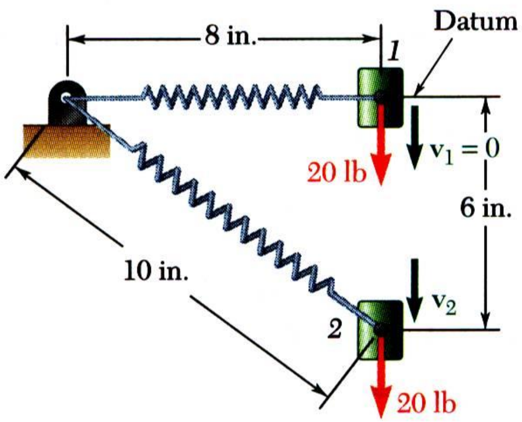
A 20 lb collar slides without friction along a vertical rod as shown. The spring attached to the collar has an undeflected length of 4 in. and a constant of 3 lb/in.

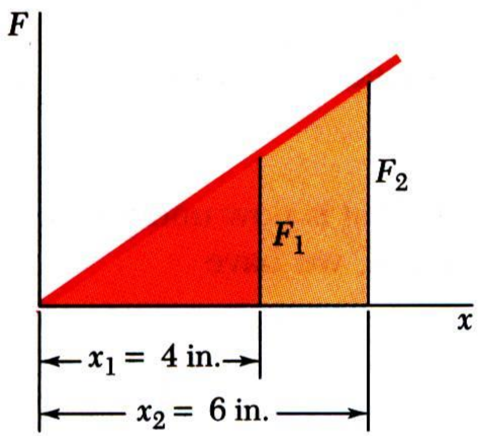
If the collar is released from rest at position 1, determine its velocity after it has moved 6 in. to position 2.

SOLUTION:

- Apply the principle of conservation of energy between positions 1 and 2.
- The elastic and gravitational potential energies at 1 and 2 are evaluated from the given information. The initial kinetic energy is zero.
- Solve for the kinetic energy and velocity at 2.

Sample Problem 13.6





<u>SOLUTION</u>:

- Apply the principle of conservation of energy between positions 1 and 2.

Position 1: $V_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(3\,\text{lb/in.})(8\text{ in.} - 4\text{ in.})^2 = 24\text{ in.}\cdot\text{lb}$

$$V_1 = V_e + V_g = 24\text{ in.}\cdot\text{lb} + 0 = 2\text{ ft}\cdot\text{lb}$$

$$T_1 = 0$$

Position 2: $V_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(3\,\text{lb/in.})(10\text{ in.} - 4\text{ in.})^2 = 54\text{ in.}\cdot\text{lb}$

$$V_g = Wy = (20\text{ lb})(-6\text{ in.}) = -120\text{ in.}\cdot\text{lb}$$

$$V_2 = V_e + V_g = 54 - 120 = -66\text{ in.}\cdot\text{lb} = -5.5\text{ ft}\cdot\text{lb}$$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\frac{20}{32.2}v_2^2 = 0.311v_2^2$$

Conservation of Energy:

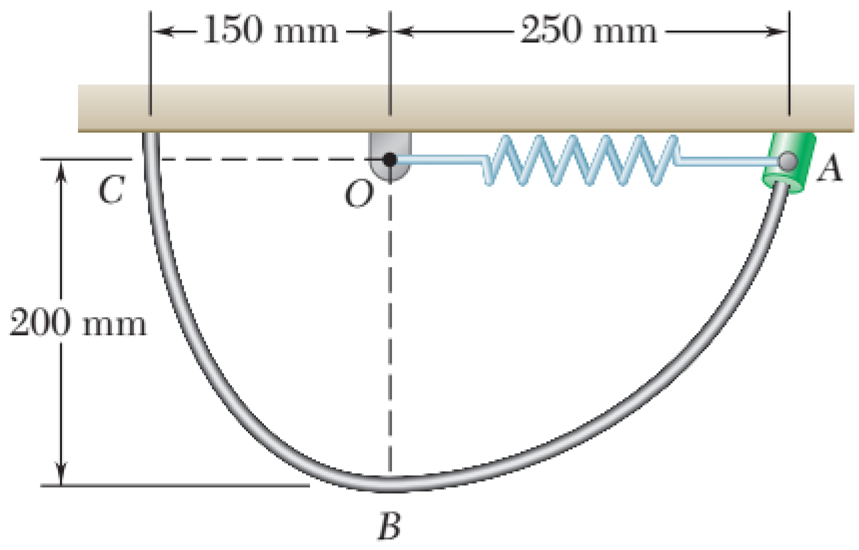
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\text{ ft}\cdot\text{lb} = 0.311v_2^2 - 5.5\text{ ft}\cdot\text{lb}$$

$$\boxed{v_2 = 4.91\text{ ft/s}\downarrow}$$

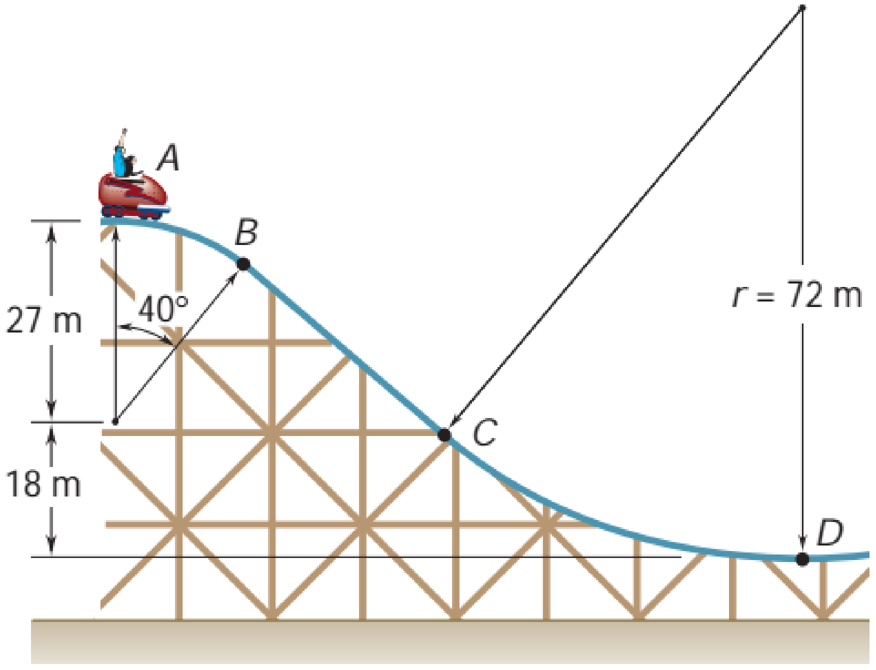
Prob# 13.64

A 2-kg collar is attached to a spring and slides without friction in a vertical plane along the curved rod *ABC*. The spring is undeformed when the collar is at *C* and its constant is 600 N/m. If the collar is released at *A* with no initial velocity, determine its velocity
(*a*) as it passes through *B*, (*b*) as it reaches *C*.



Prob# 13.70

A section of track for a roller coaster consists of two circular arcs *AB* and *CD* joined by a straight portion *BC*. The radius of *AB* is 27 m and the radius of *CD* is 72 m. The car and its occupants, of total mass 250 kg, reach point *A* with practically no velocity and then drop freely along the track. Determine the normal force exerted by the track on the car as the car reaches point *B*. Ignore air resistance and rolling resistance.



Prob # 13.68

A spring is used to stop a 50-kg package which is moving down a 20° incline. The spring has a constant k = 30 kN/m and is held by cables so that it is initially compressed 50 mm. Knowing that the velocity of the package is 2 m/s when it is 8 m from the spring and neglecting friction, determine the maximum additional deformation of the spring in bringing the package to rest.